Is It Absurd to Deny Bivalence?

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I. Introduction

In cases of borderline vagueness, it can seem intuitive to characterise certain utterances as neither true nor false on the grounds of it being unclear which, if either, of these truth values obtains. In practice, a typical language user might refrain from using such vague predicates as 'bald', 'heap', 'thin', etc. in situations where it is not clear that they apply, for example when referring to a man who is partially bald, or prefix them with a suitable modifier, such as 'partially', 'small' or 'quite'. However, according to the logical principle of bivalence, all utterances that express a proposition must possess one of two possible truth values: true or false. Furthermore, adherents to the truth-conditional theory of meaning would claim that our grasp of the meaning or 'sense' of a sentence is determined by its truth conditions; i.e. the situations under which it is true or false. This 'semantic conception of truth' (Gómez-Torrente 2006) is represented by Tarski's *Convention T*, which may be symbolised as follows:

(T1) T['P'] ↔ P

(T2) T['~P'] ↔ ~P

with T[] signifying the metalinguistic truth predicate 'It is true that …'. According to T1, the sentence 'This rose is red' is true if and only if the rose in question is actually red. But what if the rose was not red, but a borderline case of red and pink? In this case, it would not be *true* that the rose is red, but neither would it be *false*, at least not in any straightforward sense. Similarly, the negation of this sentence, 'It is not the case that this rose is red', would also appear to lack a determinate truth value. If we are to accept these linguistic

intuitions at face value, it seems that we must allow for the possibility that some utterances are *neither true nor false*, but rather possess a third or 'indeterminate' truth value, thus contravening the principle of bivalence.

II. Williamson's *Reductio*

In an influential paper entitled 'Vagueness and Ignorance', Timothy Williamson (1992) presents a *reductio ad absurdum* argument against the denial of bivalence on the grounds that it entails a denial of the Law of the Excluded Middle[1] in the metalanguage, i.e:

(W1) ~(T['P'] ∨ T['~P'])

By replacing the two disjuncts with the right-hand side of the Tarskian biconditionals, T1 and T2, this gives:

(W2) ~(P ∨ ~P)

which, by De Morgan's Law, entails:

(W3) ~P & ~~P

Such a contradiction is clearly unacceptable to the proponent of any non-classical logic that denies bivalence.[2] The root of the contradiction may be traced back to Williamson's use of classical (i.e. bivalent) logic in the metalanguage of W1. The bivalence in the metalanguage is then 'forced through' to the object language via *Convention T* with disastrous consequences. To refute the argument, therefore, one must either adopt a non-classical metalanguage or reject T1 and T2, the basis of Tarski's semantic conception of truth, or both. Given the above diagnosis, I will consider whether (a) the opponent of bivalence is committed to accepting Williamson's assumptions, and

[1] The principle that (P ∨ ~P) is true for all values of P.

[2] Except perhaps being dialetheism, which permits statements to be both true and false.

(b) whether by rejecting them they can successfully overcome the above argument.

III. Multivalence and Assertibility

Pelletier and Stainton (2003: 372) deny that LEM necessarily fails in a three-valued (i.e. true, false and indeterminate[3]) logic provided that the negation operator only ever yields determinate truth or falsity even in borderline cases, as illustrated in *Figure 1*. This is known as 'exclusion negation' on the basis that only one of the three possible truth values (i.e. determinate truth) is being denied, or 'excluded'. On this view, the second disjunct of W1 is true where P is indeterminate, and so Williamson's argument never even gets off the ground.

P	**~P**
T	F
I	T
F	T

Figure 1: exclusion negation (*ibid.*)

Furthermore, it is doubtful that both directions of the Tarskian biconditionals hold for any such three-valued logic. Although the left-to-right direction is uncontentious, since T['P'] → P is true whenever T['P'] is false, the reverse does not apply. If P is indeterminate, which is defined to mean *neither true nor false*, then it is not true that P, and so P → T['P'] is either false or indeterminate (*Figure 2*). Regardless of which of these options we take, the right-to-left direction of T1 and T2 will fail in cases of borderline vagueness, and so these statements do not constitute logical truths in the relevant multivalent logic. Thus, as Pelletier and Stainton conclude, T1 and T2 '*already presuppose*

[3] I have used the term 'indeterminate' in preference to 'definite' to avoid begging the question as to whether vagueness is a primarily epistemic, metaphysical or linguistic notion. No connection with the metaphysical notion of *determinism* should be implied or assumed.

bivalence, and so they are not the appropriate disquotational schemas for a multivalued logic' (*ibid.* 374).

T['P']	**P**		**T['P'] → P**	**P → T['P']**
T	T	|	T	T
F	I	|	T	F or I
F	F	|	T	T

Figure 2: the 'semi-standard' biconditional (ibid. 375)

Williamson (*op. cit.* 268–9) justifies his use of *Convention-T* on the grounds that any predicates (e.g. baldness) which fail to refer to sharply defined properties actually say nothing at all, and so we should refrain from using them, as per other cases of reference failure. Given that most concepts contain at least some element of vagueness, this effectively renders most utterances meaningless, thus constituting a further argument against the denial of bivalence, or else in favour of some sort of linguistic nihilism (cf. Unger 1979). However, this claim is too strong. In order for a proposition to say something, it is arguably sufficient for there to be cases in which it is *determinately* true or *determinately* false. In cases of borderline vagueness, where neither the proposition nor its negation can truthfully be asserted, its use might be considered *incorrect* or *inappropriate*, but not *meaningless* since we might, for example, legitimately assert it as a form of exaggeration, or to make a point, even though the resulting utterance would be literally false. On this view, whilst the meaning of the sentence may be fixed by its (determinate) truth conditions, its use is governed by a separate set of *assertibility conditions* — the conditions under which it can truthfully be asserted. In the case of vague predicates, these two sets of conditions come apart, leaving gaps in our assignment of determinate truth and falsity, and resulting in what Pelletier and Stainton (*op. cit.* 371) call a 'truth-value gap' theory. The notion of truth corresponding to this theory may be symbolised as:

(T1*) T[‘P’] ↔ ΔP

(T2*) T[‘~P’] ↔ Δ~P

where the Δ operator signifies ‘It is determinately true that …’.[4] Substituting T1* and T2* into Williamson’s original argument, we obtain the entirely plausible and non-contradictory conclusion:

(W3*) ~ΔP & ~Δ~P

which states that ‘it is not the case that *determinately* P, nor is the case that *determinately* not-P’ — precisely what we might expect the opponent of bivalence to say in borderline cases of vagueness (cf. Williamson 1994: 194).

IV. Higher-Order Vagueness

The introduction of the determinately operator, however, presents another problem: at what point does a proposition become determinately true or determinately false? Although we can cite paradigm cases of determinate truth and falsity (e.g. ‘snow is white’, ‘the moon is made of green cheese’, etc.), it is unclear exactly where the boundaries lie. In other words, *determinate truth is itself a vague concept*, thus giving rise to the problem of higher-order vagueness. As Williamson (2003: 8) points out, any analysis of vagueness that cannot account for higher-order vagueness is implausible (although perhaps not *absurd*) as we cannot deny that the concept of vagueness itself permits of borderline cases. Furthermore, we must be able to assign a particular meaning to the concept of determinacy that is distinct from the basic unqualified conception of truth (Williamson 1994: 194).

One approach to the problem of higher-order vagueness would be to treat the determinately operator as a sentence modifier whose truth value may itself be indeterminate. Thus the statement ‘It is determinately true that P’, symbolised as ΔP, may be neither true nor false in cases of higher-order

[4] The Δ operator effectively ‘collapses’ the indeterminacy.

vagueness (although ΔΔP or ΔΔΔP may be determinate, since multiple iterations of the Δ operator corresponding to ever increasing orders of vagueness). However, this is unattractive as it would render the determinately operator non-truth functional, undermining the notion of truth given in T1* and T2*. An alternative approach proposed by McGee and McLaughlin (1995: 229–30) is to explain higher-order vagueness in terms of higher-order instances of the non-bivalent metalanguage. On this account, determinacy is a metalinguistic concept, comparable to T[], by which the semantics of the object language are defined. First-order vagueness is governed by the set of constraints upon the object language plus the allowable models that assign truth and falsity to each of its sentences. Unlike a precise object language, for which there is only one such model, a vague language will possess many such models, each of which assigns different truth values to the various vague sentences in the language. Determinate truth and falsity correspond to the cases where all of these models agree upon a sentence's truth value (i.e. the Δ operator acts as a sort of quantifier over allowable models). Second-order vagueness is similarly defined in terms of allowable models of the *metalanguage*, with a second-order metalanguage (the metametalanguage) governing the formal semantics of the constraints of the first-order metalanguage, including the concept of determinate truth, which inherits the vagueness of terms in the object language. This yields a series of modified biconditionals such as:

(T1**) $\Delta T[\text{`P'}] \leftrightarrow \Delta P$ (*ibid.*)

each of which define the (vague) constraints upon the previous level of language, and whose vagueness is resolved at the subsequent level. This pattern may be repeated indefinitely to cope with any number of orders of vagueness without fear of contradiction — or as McGee and McLaughlin put it, '[a]s we ascend the hierarchy of metalanguages, we find vagueness all the way up' (*ibid.* 230).

V. Conclusion

Williamson's *reductio* represents a forceful attack on the denial of bivalence. However, its assumption of a classically bivalent metalanguage, along with the use of Tarski's *Convention-T*, are clearly question begging. When combined with an appropriate three-valued logic and a modified conception of truth that takes both higher-order vagueness and determinacy into account, the alleged absurdity gives way to mere complexity. We can conclude that although it is not absurd to deny bivalence, such a move loses much of the simplicity and elegance of classical logic, as well as requiring serious revision to the conception of truth represented by the Tarskian biconditionals. Whether this in itself constitutes an argument against bivalence, as Williamson (1992: 279–80) claims, remains open to question. Although there are advantages to adopting a classical approach in cases where vagueness is not an issue or can safely be ignored, the 'truth value-gap' theory is arguably a better model of our actual linguistic practices, and avoids the need for the implausible assumption that either meanings or interpretations are wholly precise, as per epistemicism (*ibid.*) and supervaluationism (Fine 1975), respectively. Instead, by treating vagueness as a positive phenomenon that is an integral part of natural language, we are forced to adopt a more complex, but arguably more sophisticated, account of truth and meaning.

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